العنوان:	TRANSIENT PERFORMANCE A TWO-DIMENSIONAL SALT GRADIENT SOLAR POND : A NUMERICAL STUDY
المؤلف الرئيسي:	Mansour, Rami Reda
مؤلفين آخرين:	El Sayed, Moustafa Mohammed، El Refaee, Mohammed(sup، SUPER)
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ABSTRACT

Solar ponds have recently become an important source of energy that is used in many different applications. The technology of the solar pond is based on storing solar energy in salt-gradient stratified zones. Many experimental and numerical investigations concerning optimum operational conditions and economical feasibility of solar ponds have been published in the last few decades. In the present study, a novel two-dimensional mathematical model that uses derived variables is developed and presented. This model utilizes vorticity, dilatation, density, temperature and concentration as dependent variables. The resulting governing partial differential equations are solved numerically by a semi-implicit finite differencing scheme to predict the transient performance of a solar pond in two-dimensional domain. The boundary conditions are based on measured ambient and ground temperatures at Kuwait city. Based on the present formulation, a computer code has been developed to solve the problem at different operating conditions. The results are compared with the available experimental data and one-dimensional numerical results. Two-dimensionality effects are found to depend mainly on the aspect ratio of the pond. A parametric study is conducted to determine the optimum pond dimensions and operating conditions.

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a	Albedo (reflection factor)	L	Layer thickness (m)
AR	Aspect ratio	p	Pressure
C	Salt concentration	Pr	Prandtl number
$\dot{C}$	Rate of generation of salt concentration	$\bar{q}$	Heat transfer vector
C_p	Brine specific heat	q_{con}	Heat transfer by conduction
d	Day of the year (January 1st=1)	q_L	Thermal extracted load (W/m^2)
Ds	Starting date for pond operation	q_{Rad}	Heat transfer by radiation
DV	Derived variables formulation	R	Dimensionless quantity
E	Internal energy	Re	Reynolds number
$\bar{\bar{e}}$	Strain tensor	S	Source term
Ec	Eckert number	Sc	Schmidt number
F	General function	t	Time from start of operation
Fr	Froude number	T	Temperature(C^o)
g	Gravity acceleration (m/sec^2)	u	Horizontal velocity component
Ha	Available solar intensity just below pond surface (W/m^2)	v	Vertical velocity component
		$\bar{v}$	Velocity vector
Ho	Average solar intensity (W/m^2)	$\overline{v}$	Average wind speed (m/sec)
$\overline{H}o$	Global heat radiation on a horizontal surface ($kWh/m^2/day$)	x	Horizontal width of the pond (m)
k	Brine thermal conductivity (W/m.K)	y	Vertical depth of the pond (m)

GREEK LETTERS

β	Dilatation	ρ	Brine density (kg/m^3)
δ	Diffusivity coefficient	Φ	Dissipation function
δ_{ij}	Kronecker symbol	θ	Dimensionless temperature
η	Bulk-coefficient of viscosity	$\bar{\bar{\tau}}$	Stress tensor
μ	First coefficient of viscosity	$\bar{\omega}$	Vorticity vector
ν	Dynamic viscosity		
ν_d	Salt diffusion coefficient		
λ	Second coefficient of viscosity		

SUBSCRIPTS

1,2,3,4	Refer to upper, gradient, lower and ground zone.	w	Water
i,j,k	Cartesian coordinate direction	g	Ground

SUPERSCRIPTS

n	Time level	^	Intermediate state
*	Non-dimensionality symbol		

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In this thesis I would like to express my deep appreciation to Dr. M. M. El-Refaee who spent plenty of time initiating ideas, supervising and discussing the present research. My appreciation to Dr. El-Refaee extends far beyond what I could ever express because he gave me the opportunity to work in this enjoyable field.

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KUWAIT UNIVERSITY

TRANSIENT PERFORMANCE OF A TWO-DIMENSIONAL SALT GRADIENT SOLAR POND - A NUMERICAL STUDY

A Thesis in Mechanical Engineering

By

Rami Reda Mansour

Submitted in Partial Fulfillment of the Requirements for the Degree of

Master of Science

December, 1994

The signatories below indicate that they have read and approved the thesis of Rami Reda Mansour.

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Solar ponds have recently become an important source of energy that is used in many different applications. The technology of the solar pond is based on storing solar energy in salt-gradient stratified zones. Many experimental and numerical investigations concerning optimum operational conditions and economical feasibility of solar ponds have been published in the last few decades. In the present study, a novel two-dimensional mathematical model that uses derived variables is developed and presented. This model utilizes vorticity, dilatation, density, temperature and concentration as dependent variables. The resulting governing partial differential equations are solved numerically by a semi-implicit finite differencing scheme to predict the transient performance of a solar pond in two-dimensional domain. The boundary conditions are based on measured ambient and ground temperatures at Kuwait city. Based on the present formulation, a computer code has been developed to solve the problem at different operating conditions. The results are compared with the available experimental data and one-dimensional numerical results. Two-dimensionality effects are found to depend mainly on the aspect ratio of the pond. A parametric study is conducted to determine the optimum pond dimensions and operating conditions.

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CHAPTER (1)

INTRODUCTION

1. INTRODUCTION

One of the relatively recent methods used in storing solar energy is solar pond. Solar energy has become one of the most important sources of energy. Applications include heating, desalination, power production and many other useful purposes.

The configuration of solar ponds (Fig. 1) are generally identified by four different zones: the upper convective zone (UCZ), the non-convective zone (NCZ), the lower convective zone (LCZ), and the ground zone (GZ).

In the upper convective zone (UCZ), a thin convective layer of low salinity water exists. The bottom brines are known as the lower convective storage zone (LCZ). The high salt concentration in LCZ necessitates a transition gradient zone to the upper fresh water zone, Fig.1. The non-convective gradient zone (NCZ) insulates the LCZ from the cooler UCZ. Once a solar pond is established, part of the solar radiation striking the surface penetrates the pond zones to reach the LCZ where it is trapped. Part of this trapped energy is stored in the LCZ and the other part is transferred back in the water by conduction or lost through the ground zone and pond sides.

The studies that investigate the physics and performance of solar ponds exist in the literature since 1960, while artificial solar ponds were built and operated since 1970. Kalecsinsky [1] (1902) was the first one to report the phenomena of solar ponds in a naturally solar heated lake in Transylvania, Hungary. He suggested the use of artificial solar ponds to store solar energy for home and industrial uses.

Researchers have conducted several analytical, numerical and experimental models to predict the performance of solar ponds. Wang and Akbarzadeh [2] studied the relation between the efficiency of a solar pond and the temperature difference between the bottom zone and the ambient. Their analysis showed that if the temperature and salt concentration dependency for thermal conductivity of water is ignored, and average values are used, small error is observed. It has also shown that if a wet soil exists, then the ground losses are serious, otherwise it is neglected. Weinberger [3] and Tabor et al [4] investigated the physics of solar ponds and calculated the annual variation in one-dimensional non-convecting systems. A one-dimensional mathematical model that simulates all pond zones and energy flux has been investigated by Panhi [5]. He examined the influence of the effect of the surface wind and the water evaporation rate on the pond efficiency. He concluded that a reduction of the evaporation rate increases the pond efficiency and the rate of heat extraction. Xu and Peter [6] examined the use of the injection method in salinity gradient establishment for solar ponds. They developed a numerical algorithm for establishing any desired salinity profile in a solar pond. Hull [7] constructed a computer model of a salt-gradient solar pond (SGSP) to verify the validity of assuming constant solar salt solution. He showed that the thermal behavior of the solar pond depends on the transparency of the non-convective insulation layer. Rubin et al [8] applied a one-dimensional finite difference model to each layer of a salt-gradient solar pond (SGSP) with a non-reflecting bottom. Keren et al [9] developed numerical and physical experiments to compare the performance of a conventional salt gradient solar pond and a salt gradient pond operating as an advanced solar pond with an increase in overall salinity near the bottom of the gradient zone. Joshi and Kishore [10] investigated the effect of the solar attenuation modeling on the performance of solar pond. The study showed that the storage temperature is generally insensitive to the modeling of the solar attenuation. A numerical model to predict the transient thermal performance of a solar pond was developed by Chang [11]. The solar energy absorbed at each level in the pond was used as an energy source term in the heat diffusion equation. Meyer [12] developed a numerical model to describe the transient behavior of the interface between the convective and non-convective regions. Salinity and temperature profiles are determined as a function of time. The model accounts for the entrainment caused by the wind generated turbulence. Cha and Schertz [13] developed a one-dimensional model to predict the temperature and concentration gradients in solar ponds. The model includes the effect of the thermal and mass diffusivities due to turbulent wind mixing and double diffusive convection. It has been concluded that the surface convective layer increases with the wind and that in the absence of wind, the double diffusive convection is capable of maintaining (UCZ) temperature. A transient model of (SGSP) has been established by Hassab, et. al. [14] to predict solar transmission, temperature and salt distribution inside the pond for any day of the year. The study has been carried out for the Arabian Gulf conditions and it shows that a pond with thicker storage zone has less temperature fluctuation between summer and winter, hence better performance. In addition, the analysis showed that major losses of solar energy occurred due to the surface evaporation and ground conduction. Also, they have predicted that a depth of 4.1 m is enough to maintain optimum design conditions (70^{o} C-90^{o}C). Most recently, El-Refaee et. al [15] developed a 1-D model that accounts for surface evaporation, wind effects, load extraction and variations of the brine physical properties with the temperature and salinity. They recommended optimum values of 1.3 m and 1.4 m for the gradient and storage zones thickness.

A parametric study of salt-gradient solar ponds of size less than 100 m^2 is presented by Kamal [16]. The study is based on a dynamic model of the pond which takes into account the variation of solar radiation, ambient temperature and the amount of heat extracted with time. The model also considered a small scale pond whose top is covered by a transparent cover. Newell [17] presented a numerical simulation which compares performances between a conventional solar pond and a proposed solar pond configuration. The proposed pond maintains a stratified storage zone below the lower convecting zone. A two-region numerical model of the injection process using axisymmetric radial diffusers has been developed by Eghneim [18] to investigate the effects of the diffuser geometry, pond size and injection conditions on the gradient modification. The model uses the initial salinity and temperature distributions to find the fluid injection flux required to bring the gradient to a condition as close as possible to optimal. Most of the work in the literature survey considered the use of one-dimensional model to simulate and analyze the performance of solar ponds. The one-dimensional model which assumes small convective motions and very slow salt diffusion gives a very close assumption and reflects almost as a closer image as possible to the real case. However, recent research works show that a one-dimensional assumption is not efficient for many practical engineering applications where the wind effects, heat losses from pond bottom and sides, and double diffusion phenomena represents an important constraints to a complete analytical study of a solar pond.

One of the most acceptable and approved two-dimensional model is the one proposed by Newell [17]. In this investigation the thermal energy performance prediction of solar ponds is accomplished first with a one-dimensional model. The decanting method of thermal energy extraction for large scale ponds was then analyzed by a two-dimensional numerical fluid dynamics scheme. The two-dimensional governing equations were presented in the primitive variables form, but the complexity of double-diffusion systems makes the progress in two-dimensional modeling problems more difficult.

In the present work, a novel two-dimensional model that uses derived variables has been developed to predict the transient performance of a 2-D solar pond. The model solves the governing equations for the unknown derived variables: vorticity, dilatation, density, temperature and concentration. Although the primitive-variables formulation (PVF) is widely used to solve the internal-thermal flow problems the present derived-variable formulation (DVF) offers more attractive features than the PVF. The dilatation, which has a more significant physical meaning, represents a "source-like" strength due to the inhomogeninty of the saline inside the pond. It is equal to divergence of the velocity vector inside the pond. The pressure and the velocity components in the PVF is replaced by the dilatation and the vorticity in the present DVF. This indeed alleviate the difficulties that arises when prescribing the pressure boundary conditions at the walls in the PVF. Mathematically, the kinematics of the problem is divided into two simple problems. A vortical incompressible flow problem (contribution of the vorticity to the flow field) and a compressible-like potential flow problem (contribution of the dilatation to the velocity field).

Moreover, the kinetics equations are all written in transportive form which indeed simplify the numerical procedures.

Since the salinity distribution is controlled in most of the real solar ponds, the problem is simplified by assuming constant salinity distribution. In this case, the kinetics of the problem is represented only by four transport equations: the vorticty transport equation, the dilatation transport equation, the energy transport equation and the density equation. Moreover, the present problem is further simplified by calculating the density from an imperical formula that relates the density with temperature and salinity. The above simplifications have accomplished a significant reduction in the kinetics computations since only two transport equations (vorticity and energy) are to be solved iteratively. The dilatation is directly computed from the continuity equation.

In the following sections, details of mathematical and numerical models, boundary conditions and solution procedure are presented. Predictions are obtained for real operational conditions for two years of simulation. Effect of two-dimensionality is studied by changing the aspect ratio of the pond. Finally, a parametric study is conducted to determine the effect of thicknesses of the pond zones on the transient behavior of the storage temperature. Discussion and comments are presented at the end of the present thesis.

CHAPTER (2)

MATHEMATICAL FORMULATION

2.1 The continuity equation for the saline
2.2 The dilatation transport equation for the saline
2.3 The vorticity transport equation for the saline
2.4 The energy transport equation for the saline
2.5 The concentration transport equation for the saline
2.6 Normalized equation
2.7 Boundary and initial conditions
2.8 Imperical relations for the saline properties

In this chapter, the mathematical formulations that govern the transient behavior of the characteristic parameters inside the pond as well as the boundary conditions are presented. In order to improve the readability, the normalization of the derived governing equations used in this chapter is omitted in the main text and is given in appendix C.

The governing equations for the saline inside the solar pond are expressed in derived variables. The derivation of each governing equation is given below:

2.1 The Continuity Equation for the Saline

The general form of the continuity equation for a variable density flow is given as:

$$\frac{\partial \rho}{\partial t} + \bar{\nabla}.(\rho \bar{v}) = 0 \qquad (2.1)$$

However, since:

$$\bar{\nabla}.(\rho \bar{v}) = \rho \bar{\nabla}.\bar{v} + \bar{v}.\bar{\nabla}\rho$$

Substituting the above equation in equation (2.1) yields:

$$\frac{\partial \rho}{\partial t} + \bar{v}.\bar{\nabla}\rho + \rho \bar{\nabla}.\bar{v} = 0 \qquad (2.2)$$

Knowing that the total derivative for the density is:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \bar{v}.\bar{\nabla}\rho$$

and defining the dilatation β as a source strength [27]

$$\beta = \bar{\nabla}.\bar{v}$$

then equation (2.2) can be written in the form of

$$\frac{D\rho}{Dt} + \rho\beta = 0$$

or

$$\frac{D\rho}{Dt} = -\rho\beta \qquad (2.3)$$

Equation (2.3) describes the continuity equation for the saline in terms of the dilatation β.

2.2 The Dilatation Transport Equation for the Saline

The general form of the fluid momentum equation is given by:

$$\rho\frac{D\bar{v}}{Dt} = \rho\bar{g} + \bar{\nabla}.\bar{\bar{\tau}} \qquad (2.4)$$

where $\bar{\bar{\tau}}$ is the stress tensor and is given by:

$$\bar{\bar{\tau}} = (-p + \lambda\bar{\nabla}.\bar{v})\delta_{ij}\bar{e}_i\bar{e}_j + 2\mu e_{ij}\bar{e}_i\bar{e}_j$$

and $\bar{\bar{e}}$ is the strain tensor and is given by:

$$e_{ij} = \begin{bmatrix} \dfrac{\partial v_i}{\partial x_i} & i = j \\ \dfrac{1}{2}[\dfrac{\partial v_j}{\partial x_i} + \dfrac{\partial v_i}{\partial x_j}] & i \neq j \end{bmatrix}$$

By expanding the divergence of the stress tensor, and knowing that $\bar{\nabla}.\bar{v} = \beta$ we get:

$$\begin{aligned}\bar{\nabla}.\bar{\bar{\tau}} &= \bar{\nabla}.[(-p + \lambda\beta)\delta_{ij}\bar{e}_i\bar{e}_j + 2\mu e_{ij}\bar{e}_i\bar{e}_j] \\ &= \frac{\partial}{\partial x_j}(-p + \lambda\beta)\delta_{ij}\bar{e}_i\bar{e}_j + 2\mu\frac{\partial}{\partial x_j}(e_{ij}\bar{e}_i\bar{e}_j) \qquad (2.5)\end{aligned}$$

where δ_{ij} is the Kronecker symbol and is defined from tensor algebra* as follows:

$$\delta_{ij} = \begin{bmatrix} 1 & i = j \\ 0 & i \neq j \end{bmatrix}$$

Then $\frac{\partial}{\partial x_j}(-p + \lambda\beta)\delta_{ij}\bar{e}_i\bar{e}_j = -\frac{\partial p}{\partial x_j} + \lambda\frac{\partial \beta}{\partial x_j}$ (i = j)

$$= -\bar{\nabla}p + \lambda\bar{\nabla}\beta \qquad (2.6)$$

---

* Appendix B-Tensor algebra

And;

$$2\mu\frac{\partial}{\partial x_j}(e_{ij}\bar{e}_i\bar{e}_j) = 2\mu\frac{\partial}{\partial x_j}\left(\frac{1}{2}(\frac{\partial v_j}{\partial x_i}+\frac{\partial v_i}{\partial x_j})\right)$$

$$= \mu\left[\frac{\partial^2 v_j}{\partial x_j \partial x_i}+\frac{\partial^2 v_i}{\partial x_j^2}\right]$$

$$= \mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y \partial x}+\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right] \qquad (i, j = 1,2)$$

$$= \mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\mu\left(\frac{\partial}{\partial x}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})+\frac{\partial}{\partial y}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})\right)$$

Since

$$\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) = \nabla^2 u + \nabla^2 v$$

$$= \nabla^2 \bar{v}$$

and

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = \frac{\partial \beta}{\partial x}+\frac{\partial \beta}{\partial y} = \bar{\nabla}\beta$$

Then

$$2\mu\frac{\partial}{\partial x_j}(e_{ij}\bar{e}_i\bar{e}_j) = \mu\nabla^2\bar{v} + \mu\bar{\nabla}\beta \qquad (2.7)$$

From equation (2.6) and equation (2.7), the expression in equation (2.5) can be written as:

$$\bar{\nabla}.\bar{\bar{\tau}} = -\bar{\nabla}p + \lambda\bar{\nabla}\beta + \mu\nabla^2\bar{v} + \mu\bar{\nabla}\beta \qquad (2.8)$$

where the first and second coefficients of viscosity (μ,λ) depend only on the temperature and on the salinity.

Now, substituting equation (2.8) in equation (2.4) yields:

$$\rho\frac{D\bar{v}}{Dt} = \mu\bar{\nabla}\beta + \lambda\bar{\nabla}\beta + \mu\nabla^2\bar{v} - \bar{\nabla}p + \rho\bar{g} \tag{2.9}$$

Taking the divergence of each term in equation (2.9) gives:

$$\bar{\nabla}.\left[\rho\frac{D\bar{v}}{Dt} = \mu\bar{\nabla}\beta + \lambda\bar{\nabla}\beta + \mu\nabla^2\bar{v} - \bar{\nabla}p + \rho\bar{g}\right] \tag{2.10}$$

Analyzing each term of the above equation separately,

$$\begin{aligned}\bar{\nabla}.\left(\rho\frac{D\bar{v}}{Dt}\right) &= \rho\left(\bar{\nabla}.\frac{D\bar{v}}{Dt}\right) + \frac{D\bar{v}}{Dt}.(\bar{\nabla}\rho) \\ &= \rho\frac{D\beta}{Dt} + \bar{\nabla}\rho.\frac{D\bar{v}}{Dt}\end{aligned} \tag{2.11}$$

$$\bar{\nabla}.(\mu\bar{\nabla}\beta) = \mu\nabla^2\beta \tag{2.12}$$

$$\bar{\nabla}.(\lambda\bar{\nabla}\beta) = \lambda\nabla^2\beta \tag{2.13}$$

$$\begin{aligned}\bar{\nabla}.(\mu\nabla^2\bar{v}) &= \mu\nabla^2(\bar{\nabla}.\bar{v}) \\ &= \mu\nabla^2\beta\end{aligned} \tag{2.14}$$

$$\bar{\nabla}.(-\bar{\nabla}p) = -\nabla^2 p \tag{2.15}$$

$$\bar{\nabla}.(\rho\bar{g}) = \rho\bar{\nabla}.\bar{g} + \bar{g}.\bar{\nabla}\rho \tag{2.16}$$

Substituting equations (2.11), (2.12), (2.13), (2.14), (2.15) and (2.16) in equation (2.10) and dividing by ρ we get:

$$\frac{D\beta}{Dt}+\frac{\bar{\nabla}\rho}{\rho}.\frac{D\bar{v}}{Dt}=\frac{2\mu}{\rho}\nabla^2\beta+\frac{\lambda}{\rho}\nabla^2\beta-\frac{\nabla^2 p}{\rho}+\bar{\nabla}.\bar{g}+\bar{g}.\frac{\bar{\nabla}\rho}{\rho} \quad (2.17)$$

Knowing that $\lambda = -\frac{2}{3}\mu$, (for most applications in compressible flows), equation (2-17) is recast into the following form:

$$\frac{D\beta}{Dt}=\frac{4}{3}\nu\nabla^2\beta+\bar{\nabla}.\bar{g}+\bar{g}\cdot\frac{\bar{\nabla}\rho}{\rho}-\frac{\nabla^2 p}{\rho}-\frac{\bar{\nabla}\rho}{\rho}.\frac{D\bar{v}}{Dt} \quad (2.18)$$

Equation (2.18) is the general form of the dilatation transport equation. Where the left hand side represents the total convective term while the first term on the right hand side represents the diffusion. The other terms are the source terms.

2.3 The Vorticity Transport Equation for the Saline

The general form of the momentum equation has been derived as in equation (2.9) in the form of :

$$\rho\frac{D\bar{v}}{Dt}=\mu\bar{\nabla}\beta+\lambda\bar{\nabla}\beta+\mu\nabla^2\bar{v}-\bar{\nabla}p+\rho\bar{g}$$

Applying the curl operator on each term of the above equation, we obtain:

$$\bar{\nabla}x\left[\rho\frac{D\bar{v}}{Dt}=\mu\bar{\nabla}\beta+\lambda\bar{\nabla}\beta+\mu\nabla^2\bar{v}-\bar{\nabla}p+\rho\bar{g}\right] \quad (2.19)$$

Analyzing each term of the above equation separately (*),

$$\bar{\nabla}x\rho\frac{D\bar{v}}{Dt}=\rho\bar{\nabla}x\frac{D\bar{v}}{Dt}-\frac{D\bar{v}}{Dt}x\bar{\nabla}\rho$$

$$=\bar{\nabla}\rho x\frac{D\bar{v}}{Dt}+\rho\bar{\nabla}x\frac{D\bar{v}}{Dt}$$

* Appendix A-Vector algebra

$$= \bar{\nabla}\rho x \frac{D\bar{v}}{Dt} + \rho\bar{\nabla}x\left(\frac{\partial\bar{v}}{\partial t} + \frac{1}{2}\bar{\nabla}v^2 - \bar{v}x(\bar{\nabla}x\bar{v})\right)$$

$$= \bar{\nabla}\rho x \frac{D\bar{v}}{Dt} + \rho\frac{\partial}{\partial t}(\bar{\nabla}x\bar{v}) + \frac{\rho}{2}\bar{\nabla}x\bar{\nabla}v^2 - \rho\bar{\nabla}x\bar{v}x(\bar{\nabla}x\bar{v})$$

Considering vorticity $\bar{\omega} = \bar{\nabla}x\bar{v}$ and $\bar{\nabla}x\bar{\nabla}v^2 = 0$, then

$$= \bar{\nabla}\rho x \frac{D\bar{v}}{Dt} + \rho\frac{\partial\bar{\omega}}{\partial t} - \rho\bar{\nabla}x\bar{v}x\bar{\omega}$$

$$= \bar{\nabla}\rho x \frac{D\bar{v}}{Dt} + \rho\frac{\partial\bar{\omega}}{\partial t} + \rho\bar{v}.\bar{\nabla}\bar{\omega} + \rho\bar{\omega}(\bar{\nabla}.\bar{v}) - \rho(\bar{\omega}.\bar{\nabla})\bar{v}$$

$$= \bar{\nabla}\rho x \frac{D\bar{v}}{Dt} + \rho\frac{D\bar{\omega}}{Dt} + \rho\bar{\omega}(\bar{\nabla}.\bar{v}) - \rho(\bar{\nabla}.\bar{\omega})\bar{v}$$

$$= \bar{\nabla}\rho x \frac{D\bar{v}}{Dt} + \rho\frac{D\bar{\omega}}{Dt} + \rho\bar{\omega}\beta \tag{2.20}$$

where $\bar{\nabla}.\bar{\omega} = \bar{\nabla}.\bar{\nabla}x\bar{v} = 0$

$$\bar{\nabla}x\mu\bar{\nabla}\beta = \mu\bar{\nabla}x\bar{\nabla}\beta = 0 \tag{2.21}$$

$$\bar{\nabla}x\lambda\bar{\nabla}\beta = \lambda\bar{\nabla}x\bar{\nabla}\beta = 0 \tag{2.22}$$

$$\bar{\nabla}x\mu\nabla^2\bar{v} = \mu\nabla^2(\bar{\nabla}x\bar{v})$$
$$= \mu\nabla^2\bar{\omega} \tag{2.23}$$

$$\bar{\nabla}x(-\bar{\nabla}p) = 0 \tag{2.24}$$

$$\bar{\nabla}x(\rho\bar{g}) = -\bar{g}x\bar{\nabla}\rho = \bar{\nabla}\rho x\bar{g} \tag{2.25}$$

Substituting equations (2.20), (2.21), (2.22), (2.23), (2.24) and (2.25) in equation (2.19) then dividing by ρ and rearranging we get:

$$\frac{D\bar{\omega}}{Dt} = \nu\nabla^2\bar{\omega} + \frac{\bar{\nabla}\rho}{\rho} x \bar{g} - \frac{\bar{\nabla}\rho}{\rho} x \frac{D\bar{v}}{Dt} - \bar{\omega}\beta \qquad (2.26)$$

Equation (2.26) is the general form of the vorticity transport equation. Where the left hand side represents the total convective term while the first term on the right hand side represents the diffusion. The other terms are the source terms.

2.4 The Energy Transport Equation for the Saline

The general energy conservation equation is written in tensor form as:

$$\rho \frac{DE}{Dt} = \tau_{ij}\frac{\partial v_j}{\partial x_i} - \frac{\partial q_i}{\partial x_i} - \frac{q_L}{L_3} \qquad (2.27)$$

where;

E is the intensive internal energy.

$\bar{q}$ is the heat transfer vector (including conduction and radiation)

q_L is the thermal extracted load.

Considering the first term on the right hand side of equation (2.27), then;

$$\tau_{ij}\frac{\partial v_j}{\partial x_i} = \left[(-p + \lambda\beta)\delta_{ij} + 2\mu e_{ij}\right]\frac{\partial v_j}{\partial x_i}$$

Following a similar analysis from section 2.2, we get:

$$(-p + \lambda\beta)\delta_{ij}\frac{\partial v_j}{\partial x_i} = -p\frac{\partial v_i}{\partial x_i} + \lambda\beta\frac{\partial v_i}{\partial x_i} \qquad (i = j)$$

But $\frac{\partial v_i}{\partial x_i} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \vec{\nabla}.\vec{v} = \beta$

Then $(-p + \lambda\beta)\delta_{ij}\frac{\partial v_j}{\partial x_i} = -p\beta + \lambda\beta^2$ (2.28)

also,

$$2\mu e_{ij}\frac{\partial v_j}{\partial x_i} = 2\mu\left[\frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right)\right]\frac{\partial v_j}{\partial x_i}$$

$$= \mu\left[\left(\frac{\partial v_j}{\partial x_i}\right)^2 + \left(\frac{\partial v_i}{\partial x_j}\right)\left(\frac{\partial v_j}{\partial x_i}\right)\right]$$

$$= \mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2\right.$$

$$\left. + \left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial v}{\partial x}\right) + \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial y}\right)^2\right]$$

$$= \mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]$$

$$= 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]$$

Defining a dissipation function Φ for the energy equation as

$$\Phi = 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial x}\right)^2\right] \quad (2.29)$$

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